NATIONAL PRESENTATION
ON GERMAN RESEARCH INTO THE DIDACTICS OF MATHEMATICS ACROSS THE LIFE SPAN – NATIONAL PRESENTATION AT PME 37

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This report is intended to give an impression of the developments on empirical research into the field of the didactics of mathematics that has been carried out in Germany and which is relevant and visible in the PME community.

First there is an outline of the current educational policy situation in Germany and its links to didactical research and its theoretical foundation (Regina Bruder). Chapters 2 (Gert Schubring) and 3 (Hans-Dieter Sill) will examine the historical development of empirical didactic research, also against the background of major social developments, characterised by the Second World War, the fall of the Berlin Wall in 1989, and Germany's reunification. In keeping with the motto of this conference “Mathematics Learning across the Life Span” chapter 4 (Silke Ruwisch, with the support of Torsten Fritzlar and recommendations from Christiane Benz, Hedwig Gasteiger, and Jens-Holger Lorenz) deals with developments in empirical research in kindergarten. and primary school. Chapter 5 (Bärbel Barzel and Rudolf Strässer) discusses aspects and tendencies in German-language empirical research into the two secondary school levels up to and including vocational training, which have also been presented so far at PME conferences. In chapter 6 Michael Neubrand takes a closer look at the „empirical turn“ - a development in didactical research in Germany in combination with large-scale studies and recent studies on teacher expertise.

All authors thank Stefan Ufer for his valuable suggestions.

1. CURRENT SITUATION OF EDUCATION POLICY IN GERMANY WITH REGARD TO EMPIRICAL RESEARCH INTO SUBJECT DIDACTICS AND ITS THEORETICAL FOUNDATION

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In the area of education, fundamental changes have been implemented in Germany in recent years. Such changes have been triggered by unsatisfactory results from comparative studies (TIMSS and PISA), particularly in respect of mathematics. For details on these developments, see Chapter 6. Focus is now on efforts to enhance quality development and quality assurance in the areas of schools and colleges. This
process turns out quite multifaceted, as responsibility for education in Germany lies with the Federal States. All activities regarding education policy are coordinated by the Standing Conference of the Ministers of Education and Cultural Affairs of the individual Federal States. For details on the German education system, see https://webgate.ec.europa.eu/fpfis/mwikis/eurydice/index.php/Germany:Overview.

With the introduction of national education standards and the establishment of an Institute for Quality Development in Education (IQB) with a view to its development and examination, a paradigm change in terms of a result-oriented governance of the education system has been initiated. This development is complemented, amongst other things, by Germany’s participation in international comparative studies and through comparative work (Büchter & Pallack, 2012; Lorenz, 2005) aimed at state-wide examination of the performance of individual schools, and also by joint reporting on the state of education on the part of the Federal Government and the States and by major research projects that are sponsored by the German Research Foundation (DFG) and the Federal Ministry of Education and Research (BMBF). Here, educationalists, psychologists and mathematics didacticians have for the first time cooperated extensively in multiannual joint projects, for details see Chapter 6.

On the other hand, professional support for teaching staff to further develop their classes and to be able to appropriately deal with the results of benchmarking, is offered far less intensively. For this reason, amongst other things, and due to the ‘risks and secondary effects’ of ‘teaching to the test’ in mathematics classes, a controversial debate on education standards and on the orientation in respect of the notion of competence is still under way amongst German-speaking subject didacticians.

In 2011, a cross-state Centre for Teacher Training in Mathematics was established in Berlin, sponsored by the ‘German Telecom Foundation’, which offers quality-assured training programmes for mathematics teachers in cooperation with numerous partners (http://www.dzlm.de/dzlm.html).

Subject didactic research with a wide range of issues is conducted, on the one hand, at universities and colleges of education which are also in charge of teacher training for all types of schools. On the other hand, there are independent institutes doing empirical research that is also dedicated to mathematics education, such as the German Institute for International Educational Research (DIPF) with a general orientation on educational governance and quality, or institutes with separate departments for mathematics education, such as the Leibniz Institute for Science and Mathematics Education (IPN) in Kiel. The Institute for Quality Development in Education in Berlin (IQB) is responsible for monitoring educational progress in German, foreign languages, mathematics and science, according to the educational Standards. Institutionally bounded fundamental research into mathematics education had already a long tradition in Germany by the Institute for Didactics of Mathematics (IDM) in Bielefeld; for further details see Chapter 2 and (Schubring, 1992).

There are numerous initiatives promoting the development of a new generation of teachers trained in subject didactics: At the federal state level, courses in subject
didactics for postgraduates are being financed, the DFG is sponsoring interdisciplinary courses for postgraduates with the integration of subject didactics, and summer schools aiming at the professionalisation of empirical research. The Society for Didactics of Mathematics (GDM) also supports young doctoral candidates with summer schools and colloquia of doctoral candidates, for instance by presenting and discussing a very broad spectrum of research methods.

In view of the developments of education policies outlined, there is an increasing demand for subject didactic expertise and joint responsibility for the changes affecting schools and teacher training is growing. Subject didacticians are involved in devising the curricula of the individual federal states as academic advisers and largely contribute to the development and content orientation of education standards for mathematics teaching from primary school up to school-leaving examinations. Moreover, subject didacticians are also involved in setting the standards in teacher training and in current further development of degree courses for teachers up to bachelor and master degree courses at colleges and universities.

The more important it is to periodically take a step back from one’s own work and to also reflect the state of the theoretical foundation of empirical research. Chapter 3 deals with the subject of respective historical developments. The following is a report on examples of the developments and trends in the theoretical foundations of research and development work on subject didactics with regard to mathematics and also on specific research methods that were brought up for discussion at PME conferences as well.

**On the development of the task concept up to notions of “working with tasks”**

In the 1960s, the usually very narrow task concept at that time met with severe criticism, cf. Lenne´(1969). “Each mathematical subdomain is, according to Lenne´, characterized by a special type of exercises, which is dealt with systematically in steps leading from the simple to the more complex forms. Lenne´ remarks critically that the use of this organizing principle, which he calls *Aufgabendidaktik*, causes the student to see mathematics more as a collection of different types of exercises, and less as an integrated whole of ideas.” (Christiansen, Howson & Otte, 1985, p.245). Nonetheless, some task concept defined in whatever way still seemed indispensable. Tasks are a key tool to concretise and implement teaching and learning concepts in mathematics. Tasks currently illustrate both education standards and “big ideas” (Kuntze et al., 2010). A differentiation between tasks in learning and performance situations is now meant to help implement innovative concepts, such as exploratory and research-oriented learning, which require more open-ended tasks (Büchter & Leuders, 2005). Hence the task concept in subject didactics with regard to mathematics has, in connection with the problem concept, gradually grown out of its use in psychology and become emancipated. Within subject didactics, a stronger target orientation from “requests to learning actions” occurred (this is the extent to which the task concept has been generalised by now), cf. also Bruder (2010). In the situation of concrete classes, the teacher, depending on his/her diagnostic competence and teaching experience, can very well assess how difficult a given task will be for most learners, but whether a
given task will constitute a specific hurdle for an individual will be determined by the individual student only. Insofar it makes sense on learning mathematics to mean by a problem an individually difficult task and thus to consider the task concept to be a generic term.

Such development of the task concept stands as a prototype for similar situations in other subject areas as well. This is not about word marks but what could hide or even be hidden behind it. Even if to date, as in the case of Hattie (2009), for example, the steps taken by him to differentiate internally are reflected in student performance with low effects only, this still does not mean that internal differentiation has “little impact” per se. The crucial point will always be according to which concepts, even though based on theory, an internal differentiation suitable for everyday use is practised by teaching staff. On this issue, there have been distinct further developments and new long-term studies over the past years (Bruder & Reibold, 2009). Finally, the question also arises as to whether further developments of already existing concepts really need to embellish themselves with new word marks or whether it might perhaps be possible to just further develop previous definitions.

Again, new developments emerge with regard to the use of tasks. Bearing in mind a background of trying to describe competence developments in students, it appears that it is hardly possible to describe development processes on the basis of an individual task only. Furthermore, it is possible to use the potential of a task in quite different ways which makes it even more important to discuss “working with tasks” as an element of professionalisation for teachers (Weideneder & Ufer, 2013; Leuders & Leuders, 2013; Bruder, 2010).

One of the trends precisely goes along the lines of describing desirable competence developments by a sequence of tasks, instead of individual tasks only. In this context, however, prior condition would be empirically verified competence development models which currently are not yet available in a form that could be operationalised (Ufer et al., 2008; Reiss et al., 2011; Reichersdorfer et al., 2012), this again demonstrates the close ties between empirical studies and their theoretical foundation. This further shows, in particular, the necessity to bring in subject didactic expertise. The field of generating competence development models relating to mathematics, which many subject didacticians consider to be one of the most important research assignments over the next years, can, by way of example, not just be left to psychometrics only but requires interdisciplinary cooperation with the subject didacticians, as within large research associations (Leuders et al., 2009; Nitsch & Bruder, 2013).

**Further development of research methods**

The grown independence of subject didactics with regard to mathematics and its increasing qualification as an academic discipline also shows in the further development of research methods. Thus, for instance, the repertory grid method after Kelly (1955) could be adapted in a way so that developments of elements of pedagogical content knowledge of teachers, as defined by Shulman (1986), can be
ascertained individually, and hence statements on the effects of teacher training and training programmes can be worded as well (Lengnink & Prediger, 2003; Hellmig, 2009; Collet et al., 2007; Kuhnke-Lerch et al., 2010), going beyond subjective acceptance statements, although the latter are of relevance as well (Szymanski & Bruder, 2012).

For comparative studies and competence modellings IRT-Skaling is also used increasingly (e.g. Nitsch et al., 2013) – but mostly in cooperation with psychologists. Video-based methods are used to the investigation by teacher competence (Hugener et al., 2006; Lindmeier, 2011; Lindmeier & Ufer, 2010; J. Neubrand, 2002).

In the next two chapters, the history of empirical research into subject didactics will be elaborated on.

2. THE HISTORY OF EMPIRICAL RESEARCH INTO MATHEMATICS LEARNING IN GERMANY IN THE 20TH CENTURY

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Empirical research into the teaching and learning of mathematics has its roots in Psychology, in particular Educational Psychology since the turn of the 19th to the 20th century. Mathematics had always been a privileged subject of Psychology for research into the process of thinking. As psychological research intensified, for example at Wilhelm Wundt’s (1832-1920) institute in Leipzig, specialized research into learning processes emerged, which is at the core of Educational Psychology – represented in particular by Ernst Meumann (1862-1915). The efforts of IMUK, the Internationale Mathematische Unterrichtskommission, the precursor of ICMI, the International Commission on Mathematical Instruction, which was founded in 1908 to reform mathematics teaching, effected that research on the Psychology of mathematics teaching entered into the focus of mathematics education itself.

The emergence of empirical research at the turn of the 20th century

Since research on students’ errors constituted a key element in the empirical turn in mathematics education since the 1970s, the search regarding history of such research focused on early publications on errors. There was even a journal, first published in 1896, in other words in the relevant time period, which had the issue of error in its title: “Die Kinderfehler. Zeitschrift für Pädagogische Pathologie und Therapie”, dedicated to Pedagogical Pathology and therapy. One its four founders and chief editor was Johannes Trüper, director of an asylum for “Heilerziehung”, the education of disabled children. Trüper (1855-1921), taught in primary schools before studying Pedagogy and Psychiatry at Jena University from 1887. In 1890, he founded an ‘asylum’ in Jena for children impaired in their development.

From the very first volume, papers described the abilities of children to count and their number concepts. A paper in the second volume lamented that teaching reckoning was
the weak spot of the schools for the mentally disabled. Volume 2 also contained an appeal from Trüper for empirical research. He argued that the initial teaching of reckoning to normal children, as well as the entire field of such instruction to “the mentally debilitated”, would remain an open question and of particular concern for teachers. He required that extensive empirical research should be undertaken on the topic as this was a necessary basis for progress. “How do the spontaneous conceptions of numbers in sane children and in those with pathological predispositions emerge and develop?”

After the fifth volume the journal was renamed Zeitschrift für Kinderforschung, continuing until its 50th volume in 1944. One of the by now more extensive papers on learning difficulties in basic arithmetic, was the first paper in 1906 by a specialist who later became the key authority on dyscalculia, Paul Ranschburg. Ranschburg (1870-1945) was born in Győr in Hungary, but as this was part of the Austro-Hungarian Empire, his work was conducted in a German cultural and scientific environment. He studied Medicine at Budapest University where he graduated as a neurologist and obtained his PhD. He founded a psychological laboratory in the institutes for Therapeutic Pedagogy in Budapest. After achieving Habilitation and acting as Privatdozent, in 1918 he was appointed professor at Budapest University.

His first paper in the journal of Kinderforschung dealt with comparative studies on normal and low-achieving students, to measure the achievement potential of both groups with respect to addition tasks (Ranschburg, 1906). Ranschburg, influenced by Meumann, also published in Experimentelle Pädagogik, a journal edited by Meumann. His 1908 paper progresses beyond the merely descriptive approach. Regarding achievements in reckoning, he discussed difficulties in learning determined by mathematical concepts themselves, namely by various operations with numbers. He asked whether it was possible to determine and to explain “die Schwierigkeiten der einzelnen Rechenarten” – the difficulties of the individual operations of calculating (Ranschburg 1908/1909, p.138). His results showed that multiplication and subtraction usually presented greater problems and went on to discuss whether the empirical results obtained would allow an easy diagnosis of reckoning abilities, in particular of learners of low ability (“Schwachbändigung”).

His main work was, however, his 1916 monograph, which coined, for the first time, the two terms in Educational Psychology, Legasthenie (dyslexia) and of Rechenschwäche (dyscalculia), which would become key issues for research into the reading process and into the learning of basic arithmetic later in the 20th century. Ranschburg emphasized the key importance of undertaking psychological experiments to gain insight into the nature of dyscalculia. It is characteristic not only of his work, but also of the entire community and the practitioners, that the basic assumption about cognitive abilities was that stable predispositions were largely innate qualities. The German terms Begabung and Anlage express these unquestioned convictions. Consequently, Ranschburg understood not only reading aptitude as a “function of a special predisposition” (Ranschburg, 1916, 18), but he also declared that reckoning was not accessible to all but was the expression of a proper Anlage (1916, 22).
normal case was, therefore, when young people had acquired, by the completed sixth year, the capacity of understanding the four operations for numbers up to twenty and could execute such operations mentally. In contrast, the minority of those who were unable to perform these operations “suffer the lack of this Anlage to calculate; this lack I am summarizing in all its degrees as the form of deficiency as Rechenschwäche or Arithmoasthenie” (ibid., IV). These learners were the pupils in the Hilfsschulen.

In the book, he emphasized more explicitly that a first kind of difficulties was the mathematical content itself. Mastering even elementary operations presupposes complicated psycho-physiological acts. Based on Assoziations-Psychologie, he declared that success depended on “Vorstellungsketten” – chains of associations – being established between problem and solution (ibid., 24). The second kind of difficulties were those originating from the Anlage of the pupil.

It presents a major result of historical research that a first monograph on the Psychology of mathematics education was published within the context of IMUK. It was, in fact, due to the indefatigable energy on the part of Felix Klein (1849-1925) that such a presentation of the state of the art was commissioned, as part of a series of reports from the German sub-committee of IMUK. Klein was perspicuous enough to perceive that psychological research had already achieved sufficient substance to deserve a monographic presentation.

Klein commissioned David Katz (1884-1953) to present this dimension. Having studied mathematics and the sciences at Göttingen University, Katz was already known to Klein. Katz changed to Psychology, obtained his PhD and Habilitation in Psychology in Göttingen. When a Psychology professor at Rostock University, he had to leave Germany in 1933, to escape racial persecution by the Nazi regime. His study for IMUK was published in 1913 as Psychology and Mathematics Instruction, in which he also reported on his own research; Meumann evaluated Katz’s book in his lectures.

Katz’s approach was significantly more comprehensive than earlier studies. While those with a pedagogical orientation dealt almost exclusively with primary grades, Katz aimed at providing insights into the process of learning mathematics based on experimental Psychology for all levels of schooling. Furthermore, Katz reported extensively on research into the development of the number concept from early childhood, in the “pre-conceptual stage” (Katz 1913, 13-16), i.e. no longer focussed only on children attending schools. His perspective was to study the entire individual development. Katz did not restrict himself to arithmetic, but included studies about the development of spatial notions, which had not played a role in earlier reported investigations since, for a long time, geometry used not to be on primary school curriculums.

His conceptual basis was no longer the Psychology of associations, but the relatively new “Differential Psychology”, which emphasized different types of representations and was thus more able to investigate differences in the cognitive development of children. Katz also discussed the perennially controversial issue of mathematical
“Begabung” – giftedness. With insight, he distinguished between giftedness for mathematics as a scientist and aptitude for learning mathematics as a school subject, while rejecting the argument that special intelligence was needed, even for school mathematics (ibid., 59).

Katz’s monograph remained the only study on the Psychology of mathematics education in the international context of IMUK.

Mathematics educators entering the field

After WW I, newly-emerging conditions enabled mathematics educators to conduct research themselves. Evolving from the revolution of November 1918 and the ensuing Reichsschulgesetz of 1920, which instituted the Pädagogische Akademien, establishing teacher training for primary schools for the first time at a higher education level. While the leading discipline was Pedagogy, chairs of teaching arithmetic and geometry methodology were created. Of the first professors, at least two are known to have been attracted by the Psychology of Mathematics Education: Gustav Rose and Ewald Fettweis (1881-1967).

While neither Experimental Pedagogy nor psychological research into the learning of mathematics seems to have continued or been followed by new approaches after World War I, the new methodology of mathematics professors received and implemented their earlier works. This is shown, for instance, by a major work on the teaching of arithmetic, written by Ewald Fettweis as a methodology for teacher training in arithmetic. This handbook, first published in 1929 and re-edited until the 1960s, also referred to the same Ranschburg and Katz publications in its second and third editions of 1949 and 1950, with the exception of two PhD theses from the Weimar Republic period. One chapter on students’ errors is relatively extensive and novel. Although claiming to be based on psychological considerations, there is no experimental evidence but rather abstract reflections, attributing errors to emotional irritations, uncontrollable causes, physical instances, inattention, etc. (Fettweis, 1949, 220-235).

Gustav Rose, after studying Mathematics and Psychology in Göttingen, i.e. in the context of Felix Klein, obtained his PhD in 1914 for an experimental study of Psychology and published in 1928 a book, which presented in a systematic manner for mathematics teachers the results of empirical psychological research, combined with and supported by own classroom observations (Rose, 1928; Bauersfeld, 2012, 75).

The new stage of research from the second half of the 20th century

A qualitatively new stage is constituted from the 1950s, with the progression from small-scale research – performed by individual scientists or, typically, by PhD dissertations, which thus resulted in piecemeal, incoherent and often contradictory results – to large-scale research, enabled by funding agencies. In fact, in 1958 a huge grant from the National Science Foundation, in the wake of the Sputnik shock, established the School Mathematics Study Group (SMSG), with Ed Begle as Director. SMSG was thus able to realize systematic and extended testing of the elaborated textbook versions in classrooms and continuously revise according to the experiences
made. In a similar vein, in 1968 Heinrich Bauersfeld received a grant of 1 million German marks from the Volkswagen-Stiftung, which enabled him to launch the important Frankfurter Project, to develop the innovative series of *alef*-textbook for primary grades, testing preliminary versions with control groups, spread over the State of Hesse (Bauersfeld, 2012, 75).

What was to become the decisive factor for progress in empirical research on the Psychology of learning mathematics was the ensuing move from quantitative methods – as propagated and practiced by Begle – to qualitative methods.

In fact, it was the methodology of *case studies*, which superseded the emphasis on studies dealing with large groups, which proved to be decisive for researching on mathematics learning understood as a social process. Bauersfeld, one of its pioneers in mathematics education, had to organize workshops at meetings of German mathematics educators, in an attempt to overcome deep-seated doubts as to whether this methodology might yield valid results (Bauersfeld, 2012, 75 f.; 80 f.). In particular, the founding of the Institut für Didaktik der Mathematik at Bielefeld University in 1973, once again due to the Volkswagen Stiftung, brought the breakthrough of research into mathematics education within the West-German community of professors for *Didaktik der Mathematik* in general and of empirical research in particular. A milestone in this process was the 3rd ICME held in Karlsruhe in 1976, when PME was founded.

3. EMPIRICAL RESEARCH AND PERFORMANCE ASSESSMENTS IN MATHEMATICS TEACHING IN THE BRD AND THE GDR

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Empirical studies in mathematic-didactic research in the BRD

In his survey of empirical teaching research in the didactics of mathematics in the BRD (Voigt, 1996) Voigt distinguishes between three phases which are linked to developments in educational sciences and which are characterised by two „turns“.

According to Voigt the first pre-scientific phase ended with the formation of didactics of mathematics as a scientific field at West German Universities in the 1960’s and 70’s. The changes, described as a „realistic turn“ led to increased efforts to verify scientific theories by means of experiments. Attempts were made to discover laws of teaching and transfer these as a technology into the hand of the teacher. Heink (1991) provides an overview of empirical research in this phase. Voigt was of the opinion that the hope placed in these experiments to overcome the theory-practice gap, had not been fulfilled (Voigt, 1996, p.384).

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1 The following observations have been reproduced in abbreviated form from Sill & Sikora (2007, p. 10-33).
2 In the new German states methodology of mathematics learning as a teaching and scientific discipline was already evolving in the time of the Soviet occupied zone 1946-49 (Borneleit, 2006).
According to Voigt the third development phase begins at the start of the 80’s. The attendant “pragmatic turn” resulted in numerous ethnographic classroom observations and interpretative case studies, with which communicative and social relationships in teaching were examined in very great detail. This „everyday research“ gave new insights into the specific effective conditions of the teaching process and led to the inclusion of further relevant disciplines, such as Linguistics, Sociology and Communication research. Interpretative teaching research evolved as a new research area.

By the end of the 80’s the methods of interpretative teaching research had already become accepted as the prevailing methods, as a survey conducted by (Hasemann & Scholz, 1990) in 1987 into the ongoing empirical research projects of the 490 members of the GDM showed. Lorenz elucidates the reasons for this development (1980, p.9). The subject of these studies in the field of teaching were subjective theories of teachers, the development and testing of textual learning aids, processes of understanding teacher instructions, and the development of mathematical and algorithmic thinking in a pilot project. The problems in mathematics teaching which were examined included studies of student errors and misconceptions, student behaviour in solving complex problems, individual basic concept, problems of the relationship of girls to mathematics and promoting mathematical giftedness. All told, it became evident that the preferred method is the case study and the explorative „small scale study“ and that the interpretative methods are predominant. With few exceptions, no other methods were used and in particular there was no research into educational effect levels.

To describe the current scientific standards in planning and conducting empirical studies on pupil performance in the framework of mathematics-didactic research in Germany, an analysis of the 1990-2004 issues of the „Journal for Mathematik-Didaktik“ (JMD) can be called on. The JMD is the predominantly German-language scientific journal, which makes the most important contribution to the development of scientific standards in the didactics of mathematics. In all, the proportion of articles with empirical studies of any kind in this journal is about 30%, whereby in the period concerned methods of interpretative teaching research predominated.

However, didactical questions, arising from everyday mathematics teaching in practice, cannot be answered with this research approach alone. Interpretative teaching research applies methods taken from ethno-methodological conversational analysis, a special discipline in sociology whose aim is, through strict empirical analysis of the actual interactions, to determine formal principles and mechanisms with which the participants at a given social event can structure, coordinate and order their own actions, the actions of others and the given action situation meaningfully in their

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3 A summary of the essays on empirical research with a short depiction of the aims of the survey, of the research methods used, the populations observed and a characterisation of the type of research can be found as an attachment of Sill & Sikora (2007).
behaviour. Mathematics teaching is not regarded as a process of developing psychological dispositions, but as „a place of subjective constructions and social constitution of mathematical meanings“ (Voigt, 1996, p.387). The task of the researcher is to „re-interpret reality as interpreted by the participants“ (Voigt 1996, p.388). Although numerous hypotheses about the possible individual structures of psychological disposition can be won with these rather phenomenological considerations, their verification and generalisation, as well as the process of development of the structures, are not however the subject matter of this particular line of research.

In a fundamental contribution on the development of the didactics of mathematics Griesel (1975, p.20) describes the „development of practicable courses“ as its most important task. Wittmann also considers the development of one’s own scientific standards possible, when one frees oneself „from the fixation on the established sciences and the scientific theories developed in their environment and attunes oneself to the particular nature of the core area of the didactics of mathematics, i.e. the construction and research into teaching including the attendant theoretical framework.“ (Wittmann, 1992, p.62).

Zech and Wellenreuther adopted these ideas and shaped the concept epistemologically of „constructive development research“ (Zech & Wellenreuther, 1992, p.143), testing it in practice in the development of materials for fractional arithmetic.

Some examples of performance assessments from 1990 until 2004 are the comparative studies in German and English into mathematics teaching (e.g. Kaiser, 1997). Performance tests on special demands were also conducted, such as stereometric competence (Schwartze, 1990, n = 226), problem-solving skills (Törner & Zielinski, 1992, n = 88), functional thinking (Kurth, 1992, n = 1000), the ability to solve linear equations (Stahl, 2001, n = 864) and stochastic thinking (Rasfeld, 2004, n = 243).

**Performance assessments and empirical research into mathematics teaching in the GDR**

Performance assessments of a single class or school and beyond always played a major role in the history of mathematics teaching in the GDR⁴. The social system in the GDR defined itself as a performance-oriented society, a meritocracy, the principle "Everybody according to his ability!" was proclaimed the basic principle of distribution which also had repercussions on educational policy. School was to lay the foundations of the high level of performance and commitment of every citizen. To inspect the performance of the school, a „comprehensive, discriminating system of centralised performance controls, analyses and control mechanisms, from giving grades, tests, centralised and local in-school extracurricular performance comparisons was created“ (Döbert & Geißler, 2000, p.16).

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⁴ The German Democratic Republic (GDR) existed from 1949 to 1990, often known in English as East Germany. Further information about the historical background see e.g. http://en.wikipedia.org/wiki/History_of_East_Germany.
In the GDR there was a close link between the performance of the pupils and that of the teachers. The teachers’ performance was seen as the main factor in pupil performance. Consequently, measures to improve the pupils’ performance were targeted at the work of the teachers. To ensure the requisite performance of the teachers there was (Döbert & Geisler, 2000, p.17):

- teacher training, centrally-regulated in content and organisation,
- a state-controlled system of mandatory further education for all teachers,
- a consequently developed system of State recognition and awards,
- a tightly-organised, broad-based system of professional support, controls and
- other forms of reviewing teaching activities.

The philosophical and epistemological foundation of these views can be described, albeit briefly, as follows: points of departure were the actual demands of society on a citizen, i.e. the entirety of the cognitive, affective and motor dispositions needed to fulfil these requirements. This resulted in a system of knowledge, aptitudes and skills which was „specified in detail“, „according to content, scope and depth“ in the curricula, as it states in the law of 1965 on the uniform socialist educational system which remained in force until the end of the GDR. The aim of teaching was seen as the complete adoption of this system. This was considered possible, in principle, for every child, if the teachers organised an appropriate, differentiated process of adoption. The conception of man was dominated by the ideal of a socialistic personality “which was attainable by all”. According to these perceptions it is obvious that the cause of the shortcomings in the knowledge and ability of the pupils could be seen above all in the work of the teachers.

Since the beginning of the 60’s a very large number of performance assessments were conducted in all subjects, particularly mathematics teaching. Between 1964 and 1974 there were over 10 000 centralised and regional teaching analyses (Döbert & Geißler 2000, p.139). In the 70’s, before the introduction of the pocket calculator, the potential and effects of its use were evaluated in 20 test classes and 20 control classes, before being introduced to all schools on the basis of tried and tested teaching materials and methodological advice. Following the introduction of new curricula in school year 1982, 6 larger empirical studies on the results of mathematics teaching were conducted and evaluated by the mathematics department of the Akademie der Pädagogischen Wissenschaften and other scientists from cooperating Universities and higher education institutions. So, for three years, tests with about 800 4th grade pupils were conducted on memory command of the basic tasks with natural numbers. Tests examined pupils’ performance in reckoning with rational numbers, their work with the pocket calculator, solving word problems as well as explaining and proving mathematical statements.

The empirical studies in the schools were not isolated independent actions merely for the purposes of collecting data. They were invariably conducted with the aim of deducing indications of essential concrete changes in the school or to evaluate the effectiveness of measures already initiated. So, as a rule, these studies were linked to
the introduction of new curricula and text books. Accordingly, the extensive empirical studies in the 80’s were referred to as stress tests for the new materials. In his retrospective article on the development of mathematics teaching in the GDR as a scientific discipline, Walsch particularly emphasises the instruction experiments and teaching analyses, which demanded the coordinated cooperation of methodologists and teachers and which led to relatively good verified findings e.g. with regard to the practicability of syllabuses or the effectiveness of teaching materials and other conclusions (Walsch, 2003).

Empirical studies played a major role in the many PhD-theses written in the GDR in the field of methodology of mathematics learning. They normally followed the paradigm: starting with an identified deficit in the teaching practice, theory-based proposals for change were drawn up and attempts were made to verify these empirically in a field experiment. Based on these, teaching materials were often created which could be made available to teachers or used in other projects.

The empirical studies were not restricted to simple inquiries into pupil performance. Besides the tests, numerous other research methods were applied to define conditions for pupil performance. These included, above all: classroom visits, talks with pupils, teachers, and leaders of child and youth organisations, Head Teachers and schools inspectors as well as the analysis of class registers, tests and notebooks. However, correlations between pupil performance and the requirements were at a very general level. For example, empirical studies in the mid-60’s ascertained that pupil performance was particularly good when the teachers focussed the attention of the pupils and their cognition on the essential subject matter, when an appropriate number of exercises and repetitions was used, when a high degree of student autonomy had developed and when the instruction achieved a particularly educational effectiveness. It was established that pupil performance was mainly dependent on the pupils’ attitude towards learning and the subject itself as well as the pedagogical and specialist knowledge and ability of the teachers, while a correlation between pupil performance and class size, the social structure of the classes and material-technical conditions could not be established. One of the main consequences from the findings of the performance assessments in the 60’s was the demand to concentrate on the essentials in teaching well as the development of pupils’ auturgy. In the 80’s increasing the soundness of basic knowledge and ability, particularly with regard to its availability and applicability, was seen as the main way to improve pupil performance in mathematics.

A large number of demands with different levels of generality were always extrapolated from the empirical results gained. For example, following the evaluation of centralised control tests in school year 1964/65 some of the conclusions formulated included (Hopfe & Neigenfind, 1964):

- insufficient attention was paid to the case distinction and the discussion of the discriminant when solving quadratic equations
- much greater attention should be paid to the teaching of logical thinking.
Most of the conclusions are just as relevant today as they were almost 50 years ago. The question arises as to the actual use of drawing up such requirements in order to change teaching.

The unexpectedly poor results of the pupils and the low level of success in the short-term improvement in performance, despite the considerable efforts made, led to disillusionment about the feasibility of changing the performance of the pupils. (Döbert & Geisler 2000, p.80). As a result, the annual nationwide control tests in the 60’s after one 4-year cycle were discontinued, as no significant improvement in pupil performance could be discerned, despite the efforts made.

In one control test in the 9th grade for example, only 7% of the pupils were able to solve the following problem (Hopfe & Neigenfind, 1965):

\[
\frac{r-s}{t} = \frac{r+s}{t}
\]

The assessment report stated that these poor results should serve as a lesson to all teachers. Most teachers underestimated the approach to teaching such an equation. The notion that the level of this task might possibly have been far too high and that, in view of the very low percentage of correct solutions, any substantial improvement could hardly be expected, is not included in the comments.

The question arises as to the actual effects of the considerable effort undertaken to guarantee high pupil performance in mathematics teaching in the GDR. Seen from within, the GDR pupil performance did not improve to any great extent in the course of the years. As the GDR did not participate in international performance benchmarking no external comparison can be made. Certain indications on the actual level attained in mathematics teaching in the GDR can be obtained from the studies of Franke and Wynands. In these performance assessments on the ability to work with variables in the 9th grade, it was shown that, in general, pupils in the GDR performed better than their counterparts in the BRD (Franke & Wynands, 1991).

Overall, an almost diametrically opposed development of empirical research in mathematics teaching in the GDR and the BRD is evident. While large-scale empirical studies and field experiments were carried out in the GDR, focussing on the quantitative assessment of pupil performance, empirical studies in the BRD concentrated increasingly on single higher-quality case studies.

Some of the personnel-related and social reasons for the development of empirical research in the BRD until the mid-90’s were:

- The roots of most didacticians lay in fields of mathematics,
- The distinction in the 60’s and 70’s between practice and theory was based on the separation of the academic and practical phase of teacher training (Voigt, 1996),
- The long-lasting trauma of the failed reform of mathematics teaching in the 60’s and 70’s (Voigt, 1996),
• The lack of demands on the part of the educational administration until that time to compile teaching results. Consequently, there were no regular centralised performance reviews in most states.

The ignorance in both parts of Germany of the developments on the other side is regrettable. In scientific publications in the GDR dealing with mathematics learning, developments in the BRD were not discussed. And, even today empirical studies in the GDR are hardly received.

4. RESEARCH IN MATHEMATICS EDUCATION FOR EARLY CHILDHOOD AND PRIMARY SCHOOL IN GERMANY

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Early childhood as a mathematics education research domain

Although it has long been acknowledged that children start mathematical thinking and develop mathematical competencies at the very beginning of their lives, early childhood is still a quite young research domain in mathematics education in Germany (cf. Peter-Koop & Scherer, 2012). This particular situation emerged in Germany due to a relatively strict segregation of kindergarten pedagogy, with its non-academic background of learning by apprenticeship, on the one hand and school education, with its academic background, on the other. This separation was deepened by different focal points: whereas kindergarten pedagogy emphasized the development of the child’s personality through free play, school education was dominated by guided learning arrangements.

But things have changed in the past 10 years. In all German states curriculum documents for early childhood have been formulated and implemented in kindergarten practice (age 3-6). These curricula all include the development of mathematical competencies. Nevertheless, there is still great uncertainty among the kindergarteners who have not learnt to take responsibility for children’s cognitive development in special areas.

Nonetheless, early mathematics learning is a very active research area in Germany. In most cases mathematics education researchers which specialize in primary education have expanded their interests to include the early years. The following issues of interest can be highlighted:

- Construction and evaluation of learning environments in intervention studies: In the tradition of mathematics education as a design science (see following section) mathematically-rich situations are constructed with respect to the competencies and

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5 The following section has been written with the support of Torsten Fritzlar (University of Halle-Wittenberg) and recommendations from Christiane Benz (University of Education Karlsruhe), Hedwig Gasteiger (Ludwig-Maximilians-University Munich) and Jens-Holger Lorenz (University of Education Heidelberg).
interests of young children and to strengthen the continuity of learning trajectories over the different institutionalized learning settings (e.g. Wittmann, 2004). In addition, some researchers try to combine traditional kindergarten pedagogy with the requirements of mathematics education research. In this sense, Gasteiger is looking for natural settings which allow children to deepen their mathematical experiences, without being confronted with a special training program (cf. Gasteiger 2012). This design-research is embedded in intervention study programs with typical pretest-posttest designs and control of the main variables, as Gasteiger did with playing situations.

- Teaching-learning processes in all mathematical content areas. For a long time, interest in early mathematics learning was restricted to counting and learning numbers. Although early arithmetic is still an important field of research (cf. Benz, 2013), more mathematics education researchers are also trying to foster other content areas e.g. the ability to operate with mathematical pattern and structure (Lüken, 2012), the comparison of length (Zöllner & Benz, 2013) or the interrelation of spatial structure processes and enumeration (Reinhold et al., 2013) are examples internationally available.

- A third concentration of activities can be seen in the professional surrounding of kindergarten children. Kindergarten educators, as the professionals of these new demands, stand in the focus with their anxieties and the problem of a non-academic background (cf. Benz, 2012). But also the families and their special interest in the early promotion of their children are part of research in Germany (e.g. Acar, 2011). Besides searching for the status quo efforts are being made to professionalize adults dealing with young children in order to sensitize them to the conditions which allow mathematical experiences within the normal settings of their lives (cf. Gasteiger, 2012).

**Mathematics learning in primary school as a research domain**

The number of researchers interested in and specializing in mathematical thinking and development at primary level is increasing in Germany, a fact best reflected in the number of PhD theses in this field. Most of these studies are designed individually without being involved in larger research groups. During the past 10 years these young researchers have become more visible internationally by attending international conferences such as PME, CERME and ICME.

Early algebraic thinking (Fritzlar & Karpinski-Siebold, 2011), statistics, probability and other stochastic problems (Reiss et al., 2011; Lindmeier et al., 2012; Lindmeier & Reiss, 2013), the adaptive strategy use of 3rd graders solving multi-digit addition and subtraction (Heinze & Lipowsky, 2008; Grüßing et al., 2013) and spatial abilities and their impact on mathematical thinking (Niedermeyer & Ruwisch, 2013; Plath & Ruwisch, 2012; Reinhold et al., 2013) are typical content areas which have been or will be presented at PME.

Despite special mathematical contents, interesting general topics should also be mentioned. For example Rezat (2006; 2008) investigated the use of textbooks and
developed a theoretical framework for analyzing these situations. Besides these single projects, three main research strands will be mentioned here. Although they are presented separately, they are tightly interwoven by their underlying principles.

**Design-based research at primary level**

The understanding of mathematics education as a design science was established by Wittmann and his colleagues (cf. RF of PME 2001). It is characterized by a systemic view, fostering a continuous improvement through the processes of designing, implementing and evaluating (cf. Wittmann, 2001). This process contains different elements. On the one hand, the construction of substantial learning environments which are mathematically rich and designed around long-term curricula is at the heart of this research domain. On the other hand, reconstructive processes were undertaken which focus empirically on special aspects of teaching-learning processes, either on learners, teachers or the learning situation.

Nowadays, all these processes are combined in larger projects. Selter and colleagues have built internet portals especially for teacher training. KIRA presents a lot of material which show that “children calculate differently”. Documents of children’s construction of mathematics meaning and video clips about solving processes, as well as interview situations, help teachers and teacher educators to improve mathematics processes in the classroom (cf. http://www.kira.tu-dortmund.de). In a subsequent project these materials were combined with materials for teacher training to foster mathematics teaching and learning in primary schools as a whole (http://www.pikas.tu-dortmund.de).

**Reconstruction of interactive mathematical knowledge of primary students**

In his major talk in PME 1999 Steinbring outlined the key components of mathematical knowledge constructed interactively in the classroom. Analysing the epistemological and communicational conditions of mathematical knowledge in this social view, he stressed the function of signs as the connecting elements between mathematical knowledge and mathematical communication. In their work Steinbring and his colleagues reconstruct patterns of interactive mathematical knowledge construction (e.g. Steinbring, 2005; Söbbeke, 2005; Söbbeke & Steinbring, 2009). Teachers were also involved in these processes of reconstructing the key elements of socially constructed mathematical knowledge. They reflected on children’s discourse in small groups and their own interventions in such situations (Nührenbörger & Steinbring, 2007). Teachers took also part in a training project to advance their diagnostic competencies which in this paradigm also means changing their own communication style from being directive and trying to transfer mathematical knowledge to investigating the child’s views and ideas of mathematics and giving appropriate support (Bräuning & Steinbring, 2011). All these communicative situations themselves were the subject of further epistemological reflection e.g. to reconstruct communicative conditions of constructing didactical mathematics knowledge.
Mathematics learning in interaction and communication in primary classrooms

Starting with an interpretative sociological paradigm for discussion and interaction in mathematics classrooms, in the 1990’s Krummheuer formulated the first elements of an interactionist theory of mathematics learning (e.g. Krummheuer, 1999). Looking at the interaction of the social – the classroom interaction – and the individual – the child’s mathematics learning –, Krummheuer and his colleagues stress the importance of collective processes of argumentation and participation (e.g. Krummheuer, 1999; 2013; Brandt & Höck, 2012). In many micro-sociological studies the researchers collected large amounts of data on mathematical situations in which different combinations of people (students and students, students and teachers, children and parents) interact and construct a common understanding of the situation and its needs. In recent years, the research group has also included early mathematics learning (see RF 2 on this PME). In a process of a permanent reciprocal influence between theoretical reflection and empirical analysis this research approach attempts to formulate an interactional theory of the development of mathematics thinking: the concept of “Interactional Niche in the Development of Mathematical Thinking” (NMT) presents the status quo (Krummheuer, 2012, 2013).

Diagnostic approaches and supporting strategies

Diagnostic competencies of teachers and their ability to react in an appropriate way to support mathematics learning – these crucial elements in the teaching-learning process in Germany have already been mentioned. In a wide sense they are focused on every child: learning problems should be recognised as well as special giftedness or the actual status of mathematics learning.

Early mathematics learning and its failure

There is a long tradition in Germany of identifying and supporting children with special problems in mathematics learning which ensued mainly from one-to-one-situations in special university faculties (e.g. Lorenz at the University of Heidelberg or Schipper at the University of Bielefeld). Nevertheless, the following changes are noteworthy:

‘Developmental dyscalculia’ and learning difficulties in mathematics are seen as more important:

- by the governance: As a result, developmental dyscalculia became a compulsory part of the curriculum in teacher education in most German states;
- by the developers of tests and other diagnostic materials: more standardized and more content-specific tests for different ages of children have been developed but are published in German (cf. collected papers in Hasselhorn et al., 2013);
- by specialists in early childhood: in research there is a tendency to investigate the early numerical understanding and to search for mathematical precursors (cf. Krajewski & Schneider, 2009).
Mathematically-gifted young children

In a pioneer study Käpnick (1998) worked out theoretically and confirmed empirically specific characteristics of mathematically-gifted third- and fourth-graders. Many of these indicators have been studied intensively in several in-depth analyses in recent years; also their transferability to even younger children is currently being investigated (cf. Fritzlar, 2013 with many references).

As in many parts of the world, girls of primary school age are underrepresented in mathematical education areas. In his PhD project Benölken identified “hypothetical particularities” of mathematically-gifted girls and drew conclusions to explain this phenomenon of underrepresentation (cf. Benölken, 2012 for a summary).

Central but controversially discussed is the question, whether mathematical giftedness is “an expression of specific cognitive characteristics or [if it is] due largely to high general intellectual abilities?” (Wieczerkowski & Prado, 1993, p. 443) Nolte has been fostering mathematically-gifted third- and fourth-graders for more than 12 years. In this context she has collected data from about 1,700 children of the talent search process including the results of intelligence tests (CFT 20 or CFT 20 R) and a special math test. Her analysis shows a significantly lower correlation for students with very good results in the math test than for the whole group (Nolte, 2012). On the one hand, a decrease in correlation is partially expected due to the increasing selection of the tested group. On the other hand, it also demonstrates that mathematical giftedness cannot actually be deduced from a high IQ.

5. RESEARCH IN MATHEMATICS EDUCATION FOR SECONDARY SCHOOLS IN THE GERMAN SPEAKING COUNTRIES

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Introduction

What follows is an attempt to describe research into Mathematics Education (or Didactics of Mathematics) in German language, which is relevant and visible in the PME community. In order to arrive at this, we tried an inductive approach by looking into the PME proceedings from 2008 to 2012, identified contributions from German speaking colleagues and clustered the contributions in the best way we can see. It is obvious that we do not mention all contributions and all research topics from German speaking colleagues, but only describe those papers which fit into some rather well defined clusters as most relevant and exemplary fields of German research. German speaking Didactics of Mathematics - even for secondary mathematics teaching and learning - is more diverse, but we hope to have caught and to describe the most pertinent research relevant for the PME audience.
Subject matter didactics (big ideas & Grundvorstellungen)

The German speaking Didactics of Mathematics is known for what is often called “Stoffdidaktik” (or “subject matter didactics”). Stoffdidaktik is “an approach to [...] mathematics education and research on teaching and learning mathematics (i.e. didactics of mathematics), which concentrates on the mathematical contents of the subject matter to be taught, attempting to be as close as possible to disciplinary mathematics. A major aim is to make mathematics accessible and understandable to the learner” (for the definition and translation see Sträßer 2013 in the Springer encyclopaedia of mathematics education). Even if at present, this tradition is not as strong as it has been in the 20th century; some contributions from the German speaking didactics are still interested in a detailed analysis of the (mathematical) knowledge to be taught. Two central aspects of Stoffdidaktik are highlighted in current developments - both of them strongly concretizing the aim of giving meaning to mathematical concepts: “Big ideas” as common threads through all mathematical areas enabling orientation, linking and anchoring of knowledge (such as using representations, proofing) and “Grundvorstellungen” as basic ideas or images of individuals concerning a certain mathematical concept (such as the different roles of a variable as a placeholder or undetermined).

With an epistemological classification for Geometry, Girnat 2009 shows that subject matter didactics can easily lead into broader, especially epistemological issues. In some sense, the concept of “big ideas” (or fundamental ideas) used by Kuntze et al. 2011, Rezat 2012 and Vohns 2009 is also in line with subject matter didactics. Sometimes continuing Bruner’s general interest in fundamental ideas of scientific disciplines, these authors try to find a characterisation of Mathematics as a scientific discipline, which could be helpful for the choice of subject matter taught and to be learned in mathematics education as classroom activity.

If we look into recent papers in PME conferences, statistical (or stochastical) ideas are closely treated (see e.g. Eichler, 2009, Kuntze et al., 2010). Prediger (2009) and Prediger&Schink (2009) look into Arithmetic and the solution of word problems. Geometry is another content area, which is in the focus of didactical research in Germany and presented in the PME context (e.g. Eichler et al., 2011; Girnat, 2009; Ufer, 2011 and some technology related research, see below). Bausch, Bruder & Prescott 2011/2012 show a more global interest in mathematical pedagogical content knowledge (“MPCK” as it is called by Blömeke et al., 2011, linking this to the widespread classification of teacher knowledge introduced by Shulman).

Other authors use the theoretical background of subject matter didactics - esp. Grundvorstellungen and big ideas for design research by developing tasks and learning environments to concretize the implementation in classrooms and by research about learning and teaching routes (Prediger & Zwetschler, 2013; Hußmann et al., 2011).

The completion for all relevant topics in school mathematics is still an ongoing process of design research.
Proof and Argumentation

There is no doubt that proof is a fundamental idea for the discipline Mathematics and its teaching and learning (for an illustration see the ‘International Newsletter on the Teaching and Learning of Mathematical Proof’ at http://www.lettredelapreuve.it/). It has also been a major issue in the PME community since long. We just mention the research forum on proof in the 2010 PME meeting (see Boero et al., 2010 and the reaction by Camargo et al., 2010). For the German speaking community of Didactics of Mathematics, we can even speak of a distinct cluster of researches into this issue. This research cluster started in connection with studies on multiple representations (especially for spatial Geometry, see Reiss, 1999 at PME 23) developed into a research on (geometrical) proof (see Reiss, Klieme & Heinze, 2001 at PME 25) and also made use of international studies on proof (like the study from Healey and Hoyles, first presented by Healey at PME 24 in 2000). It gradually developed into a study on how individuals actually learn proof and argumentation (see e.g. Ufer, Heinze, & Reiss, 2008; Reiss, Klieme & Heinze, 2001; Klieme, Reiss & Heinze, 2003). In addition to this, Heinze, Reiss & Gross (2006 at PME 30) and Kuntze (2008, from the same research group) - by means of an intervention study - looked into ways of developing proof competencies of secondary students. Reichersdorfer et al. (2012) is another example of an intervention study to foster mathematical proof and argumentation.

For a research based understanding of mathematical proof, research into the structure and phases of proof seem to be more important. Klieme, Reiss & Heinze (2003) distinguished “spatial ability” (i.e. individual competencies), “declarative knowledge” (i.e. subject matter knowledge from geometry) and “methodological knowledge” as components of an ability to prove mathematically. This string of studies generated a phase model of proving described in a paper by Heinze & Reiss (2004), which in terms of phases follows a six step model proposed by Boero (1999) in the above mentioned International Newsletter. Later, Ufer, Heinze & Reiss (2009) linked proof competencies with mental models and distinguish phases of the development of a proof.

A look back into this cluster of studies on mathematical proof shows a development, which is also visible on an international level: Often starting from a narrow concept of mathematical, if not formal proof, the research activities are forced to broaden the object of study to include mathematical - and even everyday - argumentation to adequately analyse the proof phenomenon. By its own momentum, research into proof tends to generalise the object of study to proof and argumentation in a broader sense than formal mathematical proof.

Here is still a field of research necessary to deepen the understanding about the role of individual prerequisites in content knowledge, domain-specific heuristics, general problem solving skills and especially research concerning the beliefs about everyday and mathematical argumentation and proof.
Modelling

Mathematical modelling as the linkage of mathematical models to clearly stated circumstances in the real world is a very strong research focus in Germany since the eighties, when Blum and Kaiser promoted this topic as an important issue in mathematics teaching and learning and initiated an ongoing tradition of scientific exchange. Following an idea of Pollak (1979) they established a widely accepted cycle to describe mathematical modelling which is still being developed to define concisely the modelling competency (e.g. Blum & Leiss, 2007). The steps are both stages in the solving process and potential students’ barriers.

To understand students’ mental barriers and burdens the modelling process is investigated from different perspectives. Borromeo-Ferri (2007) analysed the individual learning routes through the modelling cycle from a cognitive perspective regarding whether there is an influence of preferred representations. Also the role of metacognition and metacognitive strategies is emphasized when reflecting the transition between the phases of the modelling cycle. (Kaiser, 2011; Kaiser & Maaß, 2007). The importance of modelling tasks is confirmed by further aspects. Schukajlow & Krug (2012) have shown a positive effect of modelling tasks on students’ self-regulation, self-efficacy expectations and value. Mischo & Maaß (2013) found out that beside general intelligence and reading competence the learners’ beliefs affect all steps of the modelling process. Therefore the role of beliefs fostering mathematical modelling should always be taken into account. Modelling tasks are a meaningful issue already at primary level (Peter-Koop, 2004) and especially for secondary students concerning quite different special mathematical domains. In the frame of the Traveling Salesman problem Grigoraş & Halverscheid (2008) considered, that it is a difficult challenge for students to make the transition between mathematical and real world explicit. Considerations of mathematical nature and reasoning remain mainly intuitive. Therefore it is important to integrate meta-knowledge about modelling into teaching (Kaiser & Maaß, 2007) and to support teachers with adequate professional development (Maaß & Gurlitt, 2011). To convince teachers it is necessary to understand their views and beliefs and create adequate environments for the professional development (Kuntze, 2011; Mischo & Maaß, 2013).

For the future it seems to be important to find the relations between these different areas studied in the past. For example the influence of language competencies as reading and writing for modelling has to be specified. Another example is the role of professional development activities concerning the teachers’ beliefs as a crucial point to foster students' modelling processes and beliefs about modelling.

Technology

The integration of any kind of artefacts (non-digital and digital: such as textbooks, calculators, software, learning environments) is a topic of high relevance in German mathematics didactics and most of the researchers in this field use the instrumental genesis as theoretical background. Sträßer (2009) elaborated this theory by regarding the ‘semiotic mediation’ and pointed out that it is worthwhile and important to
integrate all kind of artefacts into the discussion, as the influencing schemata and the interplay between the different artefact and mathematics can be considered as quite similar. The textbook is the most important traditional artefact for learning and teaching. Rezat (2008) investigated the utilization schemata of students using their textbook. He used an innovative methodology to make these schemata visible. Students were asked to highlight in their textbooks the parts they are focusing on and for which purpose. When specifying certain cultural-historical-utilization-schemes he realized that the schemata are not only influenced by “rules-in-action”, but should be generalized by “beliefs-in-action”. With his ambitious work he opened new methodological perspectives of research in this field. The study also shows that students do not only use the mathematics textbook when they are told to by the teacher, but that they use it in a self-directed manner, especially during activities like solving tasks and problems to prepare for assessment.

There is an ongoing debate concerning the integration of digital media not only in research but also at schools and in school administration. Fields of activities in German speaking research are mainly domain-specific studies and large-scale-studies about integrating general purpose tools (compare Drijvers et al., 2006).

Domain-specific studies with a view on technology are elaborated in the field of data and statistics (e.g. Frischeimeier & Biehler, 2013; Eichler, 2006, 2009), geometry (e.g. Hattermann, 2008), algebra (e.g. Zeller & Barzel, 2010), calculus (e.g. Weigand & Bichler, 2010), heuristic strategies (Haug, 2008, 2010) or modelling (Greefrath, 2011). Hattermann (2008, 2010) succeeded in finding special utilization schemes for 3D-geometry software, such as “system bound dragging” instead of dragging as direct manipulation in 2D, which does not work in a 3D-environment. Haug (2008, 2010) investigated the potential of a learning environment as a combination of a pre-structured DGE-file, written prompts and a clear organized interplay between paper and computer activities, which lead to a positive effect on basic problem solving strategies (such as making conjectures, identifying invariants, using auxiliary lines).

Several large-scale studies about integration of CAS (on calculators) focus different aspects. Barzel (2007) showed the importance of well-structured student-centred classroom organization. Weigand & Bichler (2010) highlighted the relationship between mathematical abilities and instrumental abilities and result in recommending one year to get students acquainted with the technology. Bruder and her research group (Pinkernell et al. 2009) have run a long-term-, large-scale comparison study in Lower Saxonia (“Calimero”) over 5 years beginning in grade 7. None of these large-scale studies result in clear statements about an immediate influence of the technology in improving in assessment degrees. The effects are more subtle and always mention further aspects of classroom management. One important aspect was investigated by Pinkernell (2010). He showed that an extensive use of CAS accompanied by specific supporting measures (mental mathematics) does not necessarily result in a weakening of basic mathematical skills without technology.
In all 16 German Federal States (“Länder”) DGS, spread sheets and function plotting tools are recommended and in some even compulsory in final examination. Recommendations and providing teachers with software is not enough to enable teachers to involve technology in their classrooms. Especially to use current software which allow the flexible use of different representations and applications it is important to provide professional development for teachers such as the open-source community and network from teachers and researchers initiated by Lavicza et al. (2010).

Concerning the communication between research and school administration we appreciate any kind of cooperation and interplay, such as Thuringia, where an expertise from a research point of view (Barzel, 2012) lead to an initiative to decide CAS use to be compulsory in the final examination (beginning in 2014).

The interplay between conceptualization, cognitive activities, and use of instruments is still a field where a lot of research still has to be done to better understand the instrumental genesis in the student’s mind concerning certain mathematical concepts (e.g. in algebra, use of representations). In the field of technology, research into teachers’ beliefs and attitudes about “traditional” and modern technology (e.g. textbooks, learning material and software) and its integration into classroom processes need further attention.

Theories of Mathematics Education

The interest in a Theory of Mathematics Education can be traced back to an initiative by Hans-Georg Steiner, who first organised a pre-conference on this issue in relation with the PME conference as early as in 1984. The interest in this issue continued to attract researchers on various occasions (like ICME and CERME congresses) and re-entered the PME conferences in 2005. The title of a research forum in the PME conference 2010 clearly indicates the form of activity, research into theories of mathematics education has now taken in the PME community: “Networking of theories in mathematics education” (cf. Bikner-A. et al., 2010). Starting from diverse theories used to underpin actual research work, this initiative tries to build a coherent net of theories in Didactics of Mathematics. Different approaches to research in Didactics of Mathematics are represented by different colleagues. The clou of the activity is the joint effort to bring together these approaches. One major methodology for this is the use of theories for analysing “old data”, which have been gathered under different approaches to find out about strengths and weaknesses of the theories under study. At the 2013 PME conference, the title of João Filipe Matos’ plenary “Trends and opportunities for research methodologies within the PME community” seems to offer a different approach by linking theories using the research methodologies. As a complement, a Working Sessions on “Collaboration on a research topic for which a research framework is already existing” seems to continue the comparative approach. Collaboration with colleagues within the community of mathematics didactics and with those from other scientific disciplines will be necessary to develop an overall Theory of Mathematics Education.
Concluding remark

The description of the current situation of research about mathematics in secondary level school is more or less a snapshot of the common threads which are visible in the PME traditions. Fields of activities in education are always influenced not only by the individual researcher’s interests but mainly by current problems, challenges, situations in schools and the topics which are brought up by school administration and ministries. Nowadays the topics which are highly demanded in German speaking countries are the inclusion of “special needs” learners, the diversity of the students’ learning processes and social backgrounds and the diagnose of strengths and weaknesses of the students. Other areas are not in the mainstream of current German speaking Didactics of Mathematics, even if they had been an active topic in the PME community. Vocational education may serve as an example of “Mathematics Learning Across the Life Span” - with an “early” appearance in 1989 (see Sträßer & Bromme, 1989), a continuation in the Discussion Group 9 at PME 1999 (entitled “Work-related Mathematics Education”). The recent German “MANKOBE” project on “Mathematical-scientific competencies in initial vocational training” (URL: http://www.ipn.uni-kiel.de/projekte/mankobe/index_eng.html) and the PME 2013 Discussion Group 7 on “Engineering Students’ Learning of Mathematics: Modeling Mathematical Competencies” show a recovering interest into vocational education in Germany. German speaking Didactics of Mathematics for secondary education is obviously a lively field of research relevant to the PME community.

6. EMPIRICAL STUDIES (LARGE SCALE) IN MATHEMATICS EDUCATION IN GERMANY: FOSTERING INTERDISCIPLINARY RESEARCH, AND POLICY USES

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Germany on the way to adjust itself internationally

Already in the 1960’s, international comparisons of students’ achievement came up and influenced educational research, as well as they impacted educational policies. The IEA (International Association for the Evaluation of Educational Achievement) conducted in 1964 the “First” of a series of international studies into the achievement of mathematics, later called FIMS (Husén, 1967), followed by the second such study, SIMS, in 1980-1982. Germany took the opportunity of FIMS, but not of SIMS. However, in the mathematics education community, comparative research also received critical echoes (Freudenthal, 1975), and this may be one of the reasons for the long lasting reserve against those studies.
Only in 1994, Germany decided to participate in the then “Third International Mathematics and Science Study – TIMSS⁶”. The policy reasons are summarized in the German report of the TIMSS-1994 study (Baumert et al., 1997, p 33): It is stated there that Germany, as the federal state it is, had at that time neither a nationwide comparable quality control over the educational achievement in its fairly diverse 16 independent educational sub-systems, nor was there for about 30 years any international adjustment of that achievement. Thus, TIMSS was intended to get information how efficient the German system was in the international context. But, compared to the “old” tradition of empirical research in mathematics education as described in Chap. 2, 3 of this report, things changed. The sheer size and the internationality grew to a never reached scale, and the interdisciplinary approaches were a kind of precondition to administer such studies.

TIMSS was able to deliver detailed and useful information about the international educational contexts, since it was far more extended and closer to the subjects’ inner needs, than the previous studies. TIMSS incorporated not only the classical achievement tests but had several other components which were important from the mathematics education view. E.g. TIMSS had the Japan-US-Germany Video Study, but also the so-called Case-Studies on the context of teaching and learning in some countries. Two Issues of Zentralblatt für Didaktik der Mathematik (ZDM) exhibited the special aspects of TIMSS related to mathematics education (Neubrand & Neubrand, 1999/2000). Internationally, TIMSS was vividly discussed in the community, e.g. at the international congresses. Further studies comparing the teaching of mathematics in the international context, including the German situation, then appeared (e.g. Kaiser, 1999, with a Germany – UK comparison). But still, there was some critical accompany of the structure and outcomes of international comparisons (Kaiser et al., 1999; Emanuelsson & Clarke, 2004). Some studies therefore searched for their own way, also including the German perspective, e.g. the Learners’ Perspective Study (LPS) (Clarke et al., 2006).

The pay-off for mathematics education research of international studies,

When it comes to the interpretation, one needs theoretical frames to make well founded judgments. The real pay-off of dealing with internationally comparative studies lies definitely herein. To find a common ground on which to compare achievement, teaching, learning etc. in different countries with different “cultures”, appropriate concepts have to be created. The basic idea is that any serious test in mathematics has to clear up its views of the area tested, and then to operationalize it. A good means to do it is to reflect on the test items given or the tasks and problems observed in the classroom. Thus, international studies cultivated (and equally are urged to be precise, focused, and practical) the roles, features, uses, goals, etc. of mathematical tasks. Only a broad variety of features of problems and tasks, and the reflections on the origins of

⁶ The letters „TIMSS“ express nowadays „Trends in International Mathematics and Science Study“. Germany participates in the primary mathematics section of that Study, but not in the branches devoted to secondary mathematics. The international adjustment of achievement in secondary mathematics runs in Germany over the PISA Study.
such features, can assure that test results can be interpreted, and classroom observations can be communicated, in ways that are useful for the development of mathematics teaching and learning.

Consequently, thinking about tasks becomes increasingly a key area in mathematics education research. Since tasks were always considered as central in mathematics education, not only in Germany, this research area could build upon previous research, e.g. of Bruder (1988), Christiansen & Walther (1986), Bromme et al. (1990). However, these ‘germs’ were further developed into several classification systems for test items and tasks, for various purposes, like those of J. Neubrand (2002) to detect the macro-structures in the TIMSS-Video lessons, of the framework of the German PISA-2000 supplementary study (Neubrand et al., 2001, see below), or even the classification used in the study COACTIV of teachers’ professional knowledge (see below; Neubrand, 2013 a). - In a word, beyond the results, large scale studies stimulated research in mathematics education.

Consequences drawn from TIMSS-1994 in Germany

The TIMSS-1994 results were in a sense disappointing to the German public. The weaknesses shown in the achievement data and equally in the comparison of teaching arising from the Video Study had a common kernel: It was not so much the lack of ability of the students to perform algorithms as such, that filled the discussions, but a certain lack in the deeper, conceptual understanding, including the ability to apply the mathematics learned in various contexts (Blum & Neubrand, 1998). Statements like this, however, become only possible after having enough theoretical knowledge about the characteristics of test items and tasks as the working substrate of a mathematics test or the teaching of mathematics in the classes.

As a consequence, at that time a big professional development (“design”-) program started in Germany. It was aimed at enhancing the efficiency of teaching mathematics and science at all levels, and becomes known under the acronym “Sinus” (derived from the German title of the project; cf. BLK, 1997). It was grouped around several modules. Among them are the further development of tasks and their use, towards a broader scope of tasks in lessons, but also the learning through mistakes, and the realization of cumulative and cooperative learning.

“Sinus” is still working, resting on an ample expertise (BLK, 1997), written as a policy document, but in close collaboration between the administrative people and authors from scientific fields like pedagogy, educational psychology, and the subject didactics, including mathematics education (see Chap. 5.1: Mathematics under the perspective of modern education). This interdisciplinary collaboration itself could be seen as a kind of progress which influenced further projects in research and development. Since then, collaboration in the field of instructional research is on the agenda (see Chap. 1 of this report), and is increasing. A typical example is the Swiss-German video study about teaching the theorem of Pythagoras (Klieme et al., 2005/2006).
PISA and its adaption in Germany

After the TIMSS experiences, Germany decided in 1997, i.e. in fact as one of the latest countries declaring their interest, to take part also at the then upcoming, and still in a three-years distance administered study PISA (“Programme for International Student Assessment”; see from OECD, 2001 until the latest framework OECD, 2013). The focus now changed. Instead of TIMSS which was more or less explicitly orientated at an international core curriculum, PISA aimed at to know how well the education systems prepare students for life. From the beginning the PISA study attracted the interest of mathematics educators, since the key question “What is ‘mathematical literacy’?” is apparently a mathematics education topic, prepared by many considerations far before PISA (de Lange, 1987; Jablonka, 2003; Niss, 2003).

Already for the PISA-2000 cycle Germany decided to take the option the OECD gave to the countries. An ample national supplement, taking as much test time as the international test, was administered in Germany as an addition to the PISA-2000 test. In the mathematics part, this option was based on an extra and extended framework (Neubrand et al., 2001) which aims to supplement and differentiate the international PISA framework (OECD, 1999). It conceptualizes mathematical achievement in a broad sense (see later Neubrand, 2005, 2013 b). Even stronger than the international PISA framework, it capitalizes that an achievement test like PISA –under the heading ‘mathematics literacy’ – should map mathematics as broad as possible.

Typical ways of thinking and knowing in mathematics should be present in the test items. Three “types of mathematical activities” were distinguished: (i) employing only techniques, (ii) modeling and problem solving activities using mathematical tools and procedures, (iii) modeling and problem solving activities calling for connections and using mathematical conceptions. From the cognitive and the mathematical point of view these activities realize a kind of full range of mathematical thinking, since they recognize – extending PISA – technical performance also as a part of mathematics, and they discriminate the essential modes of mathematical thinking, i.e., procedural vs. conceptual thinking (Hiebert, 1986).

The PISA-2000 results (Klieme et al., 2001) then allowed to form differentiated insights into the structure of mathematics achievement (Neubrand, 2013 b), not only on the content level (e.g. Japan shows strengths geometry, but Germany weakness), but also on the cognitive level. Esp. it was shown that according to the three types of mathematical activities a test item becomes more difficult by different features (Neubrand et al., 2002); thus, mathematics achievement testing cannot restrict itself to only a limited scope of mathematics. Even within Germany, there are striking differences in the performance of the students on the three types, we speak of “profiles” or “inner structures” (Neubrand & Neubrand, 2004; Neubrand, 2005). Specific results in the PISA mathematics test are, how concept images (Grundvorstellungen, see Chap. 5 in this report) trigger mathematics achievement (Blum et al., 2004), and that even in problems which call for modeling processes, the
necessity of formalization was a decisive predictor of the difficulty (Cohors-Fresenborg et al., 2004).

**Drawing consequences from PISA: Educational Standards**

More than after TIMSS, the international PISA results led to passionate debates about status and future of mathematical (and science) education in Germany. Some even called it the “PISA shock”. The already existing public debate on the goals of mathematics in schools, and how they can be better reached, became new dynamic.

To understand from the outside the fundamental changes after PISA it is necessary to be aware that West-Germany, the BRD, had over nearly 30 years, from the 1970s on, an ideologically heated debate on those “right” school types which per se, i.e. per organization, warrant the “best” education for all. Every debate on school issues quickly became a debate on how to organize schools. Still, there is in Germany this strong division into academic and vocational tracks, in most States from the age of 10 on (see Chap. 1 of this report); however, a new element came into the debate after PISA. The administration decided to let the schools more self-responsibility on the one side, but to formulate clear expectations on what should be accomplished at several stages of schooling. With other words, the idea of Educational Standards, so far unknown in the German debate, was brought in.

The set-up of the Standards was greatly influenced by the experiences made during construction and evaluation of the PISA studies. The standards in mathematics adopted their structure largely from the PISA frameworks. They distinguish like the international PISA framework for PISA-2003 (OECD, 2003) between content areas (conceptualized as overarching ideas, becoming “Leitideen” in the Standards) and processes (conceptualized as a list of competencies, as do the Standards). From the national PISA framework, the idea was taken that tasks can have different cognitive demands, mathematically as well as from a modeling view point (Blum et al., 2006). Content and process dimensions change smoothly with the ages the standards are for, but by and large we have argumentation, problem-solving, modeling, representation, working symbolically, and communication as the processes, and number, measures, space & shape, function, and data & probability as the content ideas. The Standards are documented (in German only) on the website of the Standing Conference of the Ministers of Education: www.kmk.org.

Furthermore, the Standards are the basis for tests which are now administered in regular time distances to representative samples. However, these tests are at low stakes to the students, but serve as feedback to the administration to control the educational progress – a quite new situation which we never had in Germany before (besides some of the performance assessments realized in the former GDR, see Chap 3 in this report).

**Drawing consequences from PISA: Research into professional knowledge of teachers**

On the academic side, also consequences from PISA-2000 were drawn. It was pointed out that knowing things better should lead to research into the knowledge, behavior,
motivation, status etc. of the teachers. Since mathematics was the major domain in the PISA-2003 cycle, there was more test time allocated to mathematics. This time was used to learn more on how students and teachers respectively think about and experience mathematics in the classes, and for a 1-year longitudinal component of mathematics in PISA-2003. But beyond these (questionnaire- and achievement) data which allowed a descriptive reconstruction of the predominant patterns of mathematics instruction, it was thought about in how far a teacher survey could provide a deeper insight into the conditions of teaching and learning mathematics (Baumert et al., 2004). This was the origin of an extra study, COACTIV (Cognitive Activation in the Mathematics Classroom and Professional Competence of Teachers), examining the structure, development, and practical relevance of teachers’ professional competence (see Kunter et al., 2013 for the concluding report).

COACTIV had to cope with big challenges. The theoretical framework for teachers’ professional competence should be apart from merely tapping personality characteristics as well as from distal indicators like professional certifications or time for initial training. It should focus directly on each, teaching as the core of the profession, and mathematics as the target of instruction. The classical distinction of Lee Shulman between Content Knowledge (CK) and Pedagogical Content Knowledge (PCK), and also other teacher education studies (cf. Neubrand et al., 2008), were the starting points, but they all had to be related to mathematics on the secondary level. In so far, COACTIV moved on new grounds, and was again an interdisciplinary venture of educational psychologists and mathematics educators (Kunter et al., 2013). As its source of the students’ data and for gaining a representative sample of German mathematics teachers COACTIV had the German PISA study in the background.

From the mathematics education view, one result seems to be central. It is indeed the professional knowledge of the teachers that influences positively the learning progress of the students. CK and PCK contribute, but it is PCK that largely mediates the cognitive structure of the mathematical learning opportunities, not directly CK. The effect goes mainly over the level of cognitive activation, measured by the characteristics of the task-set given by the teacher in the class (see details in Baumert et al., 2010). Thus, COACTIV incorporates theories about mathematical problems and tasks (as accumulated from the beginning of the large scale studies; Neubrand et al., 2013a), theories of instruction (as the concept of Cognitive Activation from Instructional Psychology; Kunter et al., 2013), and as well theories on the structure of professional knowledge. It was really an interdisciplinary project.

While COACTIV provided data on the basis of a German sample only, and focused on in-service teachers, TEDS-M (Teacher Education and Development Study in Mathematics of the IEA) did it with an international scope. TEDS-M is a comparative study of primary and secondary mathematics teacher education. It focuses how internationally primary and lower-secondary teachers are prepared to teach mathematics. Tattot et al. (2008) show the international conceptual framework, while Tattot et al. (2012) exhibit the findings. One result of TEDS-M is central from the mathematics education view: It is indeed the deliberate subject orientation in
pre-service teacher education which influences positively the professional knowledge of the teacher.

TEDS-M includes many other sub-studies, and has a fairly broad view on issues in teacher education, as is displayed in Ball et al. (2012). The international results with a special emphasis on the German situation are dealt with by Blömeke & al. (2010 a, b).

**Conclusion**

The emerging international large scale studies in the 1990-s, and especially the fact that mathematics educators increasingly were involved in these studies, had several effects on the mathematics education research, not only in Germany. The interdisciplinary collaboration extended, new methods of research opened up, to deal with bigger samples becomes possible (and desirable if the context of the research has a need for), the impact of research onto the practical development of mathematics teaching and learning was gradually widened, but deserves still a critical analysis of the specific conditions in both fields.

But still a lot is open on the research side: E.g., so long, we are far distant from seeing the structures how competencies develop in the school or even more, over the life span; we still struggle with the problem if and how one can use problems and tasks from test situations in the context of learning; we have not yet a differentiated picture of how professional knowledge of teachers can be realized in the classroom; etc.. Equally, the effects of these developments on the administrative side are also manifold: What are the consequences of the teacher education studies for the organization of teacher education, pre- and in-service? How can we derive from mathematics education research practicable courses in school, and how can we evaluate them.

But – and this is surely a side effect of the empirical studies too – international cooperation in such questions is more realistic than ever. International cooperation then could also have an additional effect: Some of our questions are too big to be tackled only in the limited national (or even university bound) space; we are challenged to enlarge international cooperation.

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